

Recursive Cube of Rings and Their Implementation in Interconnection Networks

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ABSTRACT: In this paper, we show a gathering of flexible interconnection organize topologies, named Recursive Cube of Rings (RCR), which are recursively created by adding ring edges to a strong shape. RCRs have various charming topological properties in building adaptable parallel machines, for instance, settled degree, little estimation, wide division width, symmetry, fault tolerance, et cetera. We at first break down the topological properties of RCRs. We by then show and separate a general stop free directing calculation for RCRs. Using a whole combined tree embedded into a RCR with advancement cost approximating to one, a capable communicate directing calculation on RCRs is proposed. The upper bound of the amount of message passing steps in a solitary communicate operation on a general RCR is furthermore induced.

Key Word- RCR, GS, Fault Tolerance,, CP,

logarithmically with the traverse of the system however the distance across of hypercube is close to nothing.

Starting late, various new topologies have been proposed. Taking the aftereffect of two set up topologies is an up and coming system for growing new interconnection systems. Advancement of such a thing system requires first picking a base reference, for instance, de Bruijn systems, modify exchange systems, and complete double trees. The base parts may be unmistakable. The cross consequence of interconnection systems beats standard topologies, for instance, work and hypercube in distance across, degree, and organizing size. Straight recursive systems will be systems that are conveyed by a direct rehash of the edge:

$$X_n = a_1 \cdot X_{n-1} + a_2 \cdot X_{n-2} + \dots + a_k \cdot X_{n-k}$$

I. INTRODUCTION

A Computer framework is adaptable if it can scale up its assets to oblige reliably extending execution and value ask. In a parallel PC framework with coursed memory designing, the arrangement of the interconnection organize topology is fundamental to the execution and adaptability of the framework. A general flexible system topology should organize as about as possible to the general correspondence cases of various valuable parallel applications to achieve low system inaction and high throughput.

To satisfy the adaptability need for interconnection systems, it is charming that an interconnection arrange has a settled degree, little measurement, wide cut width, symmetric hubs, and fault tolerance. In most existing interconnection organizes, these necessities are routinely in battle with each other. For example, in spite of the way that a N work and torus have settled degree, their breadths are $2N$ and N , independently (therefore, respectably far reaching). The center point level of a n -3D shape (hypercube) augments

Where $a_i, 1 \leq i \leq k$, are nonnegative whole numbers and $a_k \neq 0$. In every repeat, the subscript n compares to the measurement of the network X_n , while the parameter a_i shows the quantity of events of a lower dimensional network X_{n_i} inside the n -dimensional network. The level of direct recursive systems augments logarithmically with the system scale. Considering the extending inconvenience in format and packaging, if the level of a system develops with the processor gauge, the benefit of adaptability from taking the consequence of interconnection systems may be uncommonly diminished in sensible applications. A system with settled hub degree is in this way hugely charming. A cube of rings (COR) organize is another proposed arrange that offers an amicability among flexibility and gear overhead. A cube of ring system is produced to supplant each hub of a hypercube with a ring of a comparable size. It contrasts from cube-associated cycle in the strategy for choosing the cube neighbors of each hub. The cube of rings has a settled hub degree and little

distance across in the meantime, as will be shown later; the system measure that may be picked is incredibly limited.

In this paper, we propose another gathering of interconnection systems, named recursive cube of rings (RCR) arrange. A RCR is worked by recursive improvement on a given age seed (GS). A GS for a RCR includes different rings interconnected in a cube-like outline. It can be made by particular criteria, for instance, the alluring size of the system. RCRs have various appealing topological properties in building flexible parallel machines, for instance, settled degree, little distance across, high partition width, and symmetry. Ring, hypercube, and cube-associated cycles are outstanding kinds of the RCRs. Likewise, we show that RCR have plane property, in which each hub of a RCR organize is arranged on no short of what one cube plane. All cube planes are associated by ring planes. This property may tremendously adjust the steering calculation framework and upgrade the implanting limit. For example, we may use the symmetry and plane property of RCR to successfully realize a communicate calculation. Using an aggregate double tree introduced in a RCR with augmentation cost approximating one, we develop a beneficial communicate steering calculation with low upper bound of the amount of message passing. A general stop free steering calculation for the RCR is furthermore displayed and inspected.

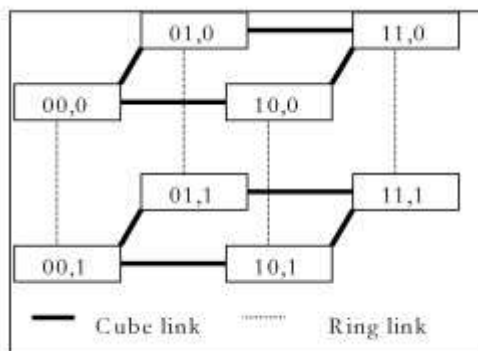


Fig 1. Generation seed GS(2, 2) for RCR.

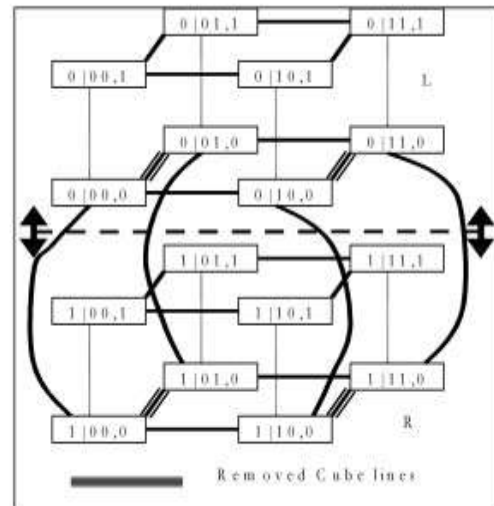


Fig .2 RCR (2,2,1) after one expansion from GS (2,2)

The paper is dealt with as tails: We at first depict the proposed RCR topology and its recursive age system in Section 2. In Sections 3, we take a gander at the topological properties of RCR. The utilization of communicate operations in light of an aggregate parallel tree, and moreover a general message steering calculation, are shown and inspected in Section 4, trailed by a conclusion in Section 5.

II. RECURSIVE CUBE OF RINGS (RCR)

A general RCR involves different rings interconnected by a couple of connections, called cube joins. The hubs inside a ring are associated by joins called ring joins. A RCR is demonstrated by RCR(k; r; j), where k is the estimation of the cube, r is the amount of hubs on a ring, and j is the amount of the improvements from the age seed.

A limit f like modulo is described for the depiction of hub locations and examination of RCRs properties. The importance of f isn't the same as the modulo because of 0 a b, which is described as takes after:

$$f(a, b) = \begin{cases} b - a, & 0 \leq a \leq b \\ a - \left\lfloor \frac{a}{b} \right\rfloor \cdot b, & a \leq b \end{cases}, \text{ where } a, b, c \in I.$$

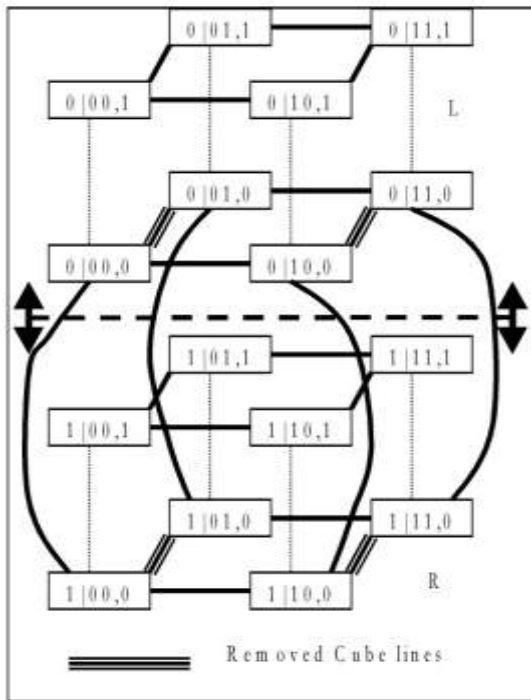


Fig. 2: RCR(2; 2; 1) after one expansion from GS

Give N_0 a chance to be the attractive number of hubs in the system, and N the quantity of hubs in the created RCR organize. We may choose a coveted estimation of r , which thus decides the estimation of j , with the end goal that N is nearest to N_0 as per the accompanying equation:

$$r = \begin{cases} \left\lfloor \frac{N'}{2^{k+j}} \right\rfloor & N > N' \\ \left\lceil \frac{N'}{2^{k+j}} \right\rceil & N \leq N' \end{cases}$$

Fig. 1 delineates an age seed GS(2; 2). Fig. 2 demonstrates a RCR(2; 2; 1) got by one development from age seed GS(2; 2), and Fig. 3 demonstrates a RCR(2; 2; 2) got from one more extension from RCR(2; 2; 1). At every extension, the quantity of hubs is multiplied and some new cube joins must be included. In the meantime, with a specific end goal to keep the steady hub degree, some cube joins must be evacuated. For instance, the hub [000; 0] and hub [010; 0] in RCR(2; 2; 1) are mapped, separately, to hub [0000; 0] and hub [0010; 0] in RCR(2; 2; 2). The cube connect ([000; 0], [010; 0]) in the RCR(2; 2; 1) is expelled.

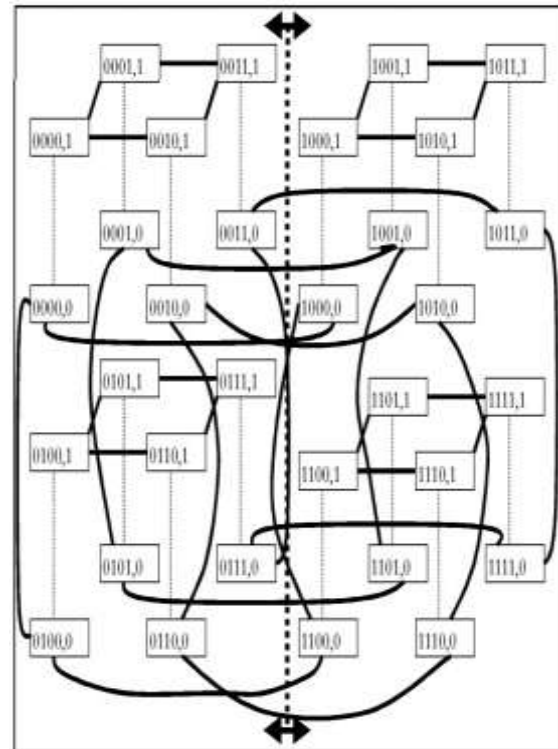


Fig. 3. The topology of RCR(2; 2; 2).

RCR(2; 2; 2), with the goal that hub [0000; 0] and hub [0010; 0] in the RCR(2; 2; 2) stay steady degree three when two new cube joins are added to these two hubs amid the extension, as appeared in Fig. 3. The calculation for developing a RCR is portrayed in Fig. 4. It is vital to take note of that, for the general cases, the real number of hubs N in a RCR system might be not the same as the coveted system measure N_0 . For instance, to manufacture a system with 20,000 hubs (N_0 : 20; 000), the span of the nearest RCR organize is 20,480, as will be clarified in Section 3.

TOPOLOGICAL PROPERTIES OF RCRS

In this segment, we analyze the major topological properties of the proposed RCR systems, for example, a RCR measure, degree, cut width, distance across, estimate coordinating property, plane property, symmetry, et cetera.

General Topological Properties

An RCR(k ; r ; j) arrange is demonstrated as a chart $G(V, E)$, where a vertex in V , G relates to a hub in the RCR organize, and an edge in E, G compares to a connection in the RCR arrange.

Plane Property

RCRs also possess a special plane property such that an RCR(k ; r ; j) can be taken as the combination of two different types of planes, cube-plane and ring-plane, to be defined below.

This property can be used to develop efficient routing algorithms.

Topology	Degree	Diameter	Bisection width
R _{CR} (2,r,j), r>2	4	$\log N - \log r + \lceil \frac{r-1}{r} \rceil - 1$	N/4
Hypercube	$\log N$	$\log N$	N/2
Cube-connected cycles	3	O(log N)	O(N/log N)
Butterfly	4	O(log N)	O(N/log N)
DeBruijn	4	O(log N)	O(N/log N)
Mesh-connected computer	4	$2\sqrt{N}$	\sqrt{N}
Torus	4	\sqrt{N}	$2\sqrt{N}$
Honeycomb torus	3	$0.81\sqrt{N}$	$2.04\sqrt{N}$

R_{CR} as a Cayley Graph

A system is symmetric if the system topology is a similar looking from any hub in the system. A symmetric interconnection system may rearrange the plan of the switches and interfaces, and along these lines decrease the cost of the systems. Cayley diagrams have been turned out to be symmetric charts. We demonstrate that R_{CR}s are Cayley diagrams, and in this way they are symmetric. The accompanying definitions are straightforwardly from.

Finally, we can construct the R_{CR} (k; r; j) in the group G based on H and the associative operator. For any node [A; b] of the R_{CR} (k; r; j) and each element from H, we derive one edge to each neighbor of [A; b] in the R_{CR} (k; r; j) as follows:

- $([A, b], [0 \dots 0, 1] * [A, b]) = ([A, b], [A, b + 1])$
- $([A, b], [0 \dots r - 1, 1] * [A, b]) = ([A, b], [A, b - 1])$
- $([A, b], [0 \dots 1, 1] * [A, b]) = ([A, b], [a_{k+j-1} \dots \bar{a}_{b_{j+1}} \dots a_0, b])$
- $([A, b], [0 \dots 10, 1] * [A, b]) = ([A, b], [a_{k+j-1} \dots \bar{a}_{b_{j+2}} \dots a_0, b])$
- $([A, b], [0 \dots 1 \underbrace{0 \dots 0}_k, 1] * [A, b]) = ([A, b], [a_{k+j-1} \dots \bar{a}_{b_{j+k}} \dots a_0, b])$.

Therefore, R_{CR} (k; r; j) is a Cayley graph. Then, according to, we conclude that the R_{CR} (k; r; j) is symmetric.

MESSAGE ROUTING IN R_{CR}

Productive steering calculations are basic for any interconnection systems. In this area, we introduce productive unicast and communicate message directing calculations for R_{CR} systems. We will accept that wormhole exchanging method is received in the R_{CR} systems. Virtual channels will be acquainted with evade the halt.

Unicast Communication

The fundamental thought of the directing calculation is like the notable e-cube steering calculation for paired cubes. It is demonstrated that every hub of a R_{CR} (k; r; j) is on sure CPs. On account of k 2, every hub is found just on a solitary CP. In one CP, the addresses [A; b]s of hubs demonstrate a standard difference in bit examples with the end goal that a similar k bit positions in An of every hub contrast and alternate bits continue as before. A fitting neighboring cube plane can be picked in the path like the e-cube directing. The leave hub of such a picked CP is the hub nearest to the goal. We call such a directing calculation in Fig. 6 a bounce plane steering calculation. The bounce plane steering calculation can simply locate a most brief way from any source hub to any goal hub in R_{CR}s.

To keep the events of halt, two virtual channels are set up on a physical connection. A hub [A; b] is appointed a whole number A r b. The hubs in the system would then be able to be requested with the relegated numbers as the keys. One of the two virtual channels, meant by vc1, is utilized when a message navigates a connection in rising request starting with one hub then onto the next. The other virtual channel meant by vc2 is utilized when the message crosses a connection in slipping request, paying little respect to cube connections or ring joins. Let (a; c) or (Aa; ba; Ac; bc) mean a connection. For a cube interface, Aa varies from Ac in one piece while ba, bc. For a ring join, ba ŷ bc j 1 while Aa, Ac. It can be demonstrated that the message steering calculation appeared in Fig. 6 is sans halt.

III. CONCLUSION

We have proposed a class of new topologies for an interconnection organize, named recursive cube of rings, which are recursively built by adding ring edges to a cube. We have demonstrated that R_{CR}s have numerous alluring topological properties in building versatile parallel machines, for example, settled degree, little distance across, plane property, wide separation width, and symmetry. We have additionally displayed and examined a general halt free steering

calculation for RCRs, and built up a proficient communicate directing calculation utilizing a total paired tree inserted into a RCR with development cost approximating to one. As for incremental versatility, the proposed RCR systems may not achieve the level of adaptability of the incrementally adaptable fragmented star diagrams proposed in, in which the hole between back to back sizes can be completely erased. Be that as it may, contrasting with the other existing topologies, for example, n-star chart and hypercube, the RCR organizes clearly have better incremental versatility as appeared in Section 3. Our future work is to build up another topology in light of the RCR systems that can accomplish the level of the incremental adaptability of the fragmented diagram while saving it to be Cayley charts.

REFERENCES

- [1]. K. Efe and A.O. Fernandez, "Products of Networks with Logarithmic Diameter and Fixed Degree," IEEE Trans. Parallel and Distributed Systems, vol. 6, no. 9, pp. 963-975, Sept. 1995.
- [2]. A.L. Rosenberg, "Product-Shuffle Networks: Toward Reconciling Shuffles and Butterflies," Discrete Applied Mathematics, vol. 37-38, pp. 465-488, July 1992.
- [3]. R.B. Panwar and L.M. Patnaik, "Solution of Linear Equations on Shuffle-Exchange and Modified Shuffle Exchange Networks," Proc. 26th Allerton Conf., pp. 1,116-1,125, 1988.
- [4]. K. Efe and A. Fernandez, "Mesh Connected Tree: A Bridge between Grids and Meshes of Trees," IEEE Trans. Parallel and Distributed Systems, vol. 7, no. 12, pp. 1,283-1,293, Dec. 1996.
- [5]. K. Day and A.-E. Al-Ayyoub, "The Cross Product of Interconnection Networks," IEEE Trans. Parallel and Distributed Systems, vol. 8, no. 2, pp. 109-118, Feb. 1997.
- [6]. E. Ganesan and D.K. Pradhan, "The Hyper-Debruijn Networks: Scalable Versatile Architecture," IEEE Trans. Parallel and Distributed Systems, vol. 4, no. 9, pp. 962-978, Sept. 1993.
- [7]. S.K. Das and A.K. Banerjee, "Hyper Petersian Networks: Yet Another Hypercube-Like Topology," Proc. Fourth Symp. Frontiers of Massively Parallel Computation, pp. 270-277, Oct. 1992.
- [8]. A.S. Youssef and B. Narahari, "The Banyan-Hypercube Networks," IEEE Trans. Parallel and Distributed Systems, vol. 1, pp. 160-169, 1990.
- [9]. X. Lin, P.K. McKinley, and L.M. Ni, "Deadlock-Free Multicast Wormhole Routing in 2-D Mesh Multicomputers," IEEE Trans. Parallel and Distributed Systems, vol. 5, no. 8, pp. 793-804, Aug. 1994.
- [10]. T.J. Cortina and Z. Xu, "The Cube of Rings Interconnection Networks," Int'l J. Foundations of Computer Science, 1997.
- [11]. W.-J. Hsu, M.-J. Chung, and A. Das, "Linear Recursive Networks and Their Applications in Distributed Systems," IEEE Trans. Parallel and Distributed Systems, vol. 8, no. 7, pp. 673-680, July 1997.
- [12]. I. Stojmenovic, "Honeycomb Networks: Topological Properties and Communication Algorithms," IEEE Trans. Parallel and Distributed Systems, vol. 8, no. 10, pp. 1,036-1,042, Oct. 1997.
- [13]. S.B. Akers and B. Krishnamurthy, "A Group-Network Theoretical Model for Symmetric Interconnection Networks," IEEE Trans. Computers, vol. 38, no. 4, pp. 555-566 Apr. 1989.
- [14]. B. Alspach, "Cayley Graphs with Optimal Fault Tolerance," IEEE Trans. Computers, vol. 41, pp. 1,337-1,339, 1992.
- [15]. W.J. Dally and C.L. Seitz, "Deadlock-Free Message Routing in Multiprocessor Interconnection Networks," IEEE Trans. Computers, vol. 36, no. 5, pp. 547-553, May 1987.
- [16]. W.J. Dally, "Virtual channel flow control," IEEE Trans. Computers, vol. 3, no. 3, pp. 194-205, Mar. 1992.
- [17]. A. Barak and E. Schenfeld, "Embedding Classical Communication Topologies in the Scalable OPAM Architecture," IEEE Trans. Parallel and Distributed Systems, vol. 7, no. 9, pp. 979-992, Sept. 1996.